Advanced Game Theory
Spring 2021

Exam 1

The exam is 3 hours. Please answer as many questions as you can. Answer shortly but justify your answers and explain accurately what you are doing. If you are confused about some question statement, please explain clearly what you assume when answering.

1. State whether the statements are TRUE or FALSE. Justify your answer. In addition, define/briefly explain the underlined concepts.

   (a) A pure action is rationalizable if it survives the iterative elimination of strictly dominated strategies. (You may assume two players if you wish.)

   (b) In the following game of incomplete information, the unique Bayes Nash equilibrium is that P1 plays U if $\theta_1 = 1$ and D if $\theta_1 = -1$ and P2 plays L.

   Players' actions and payoffs are (P1 is the row player and P2 is the column player):

	L	R
U	$\theta_1, 1$	$0, 0$
D	$0, 0$	$1, 1$

   where θ_1 is either -1 or 1. P1 knows θ_1 but P2 does not. The probability that θ_1 is 1 is $p > 0.5$.

   (c) A Sequential Equilibrium need not be a weak Perfect Bayesian Equilibrium if the game has incomplete information.

2. Two firms simultaneously decide whether to enter a market. Firm i's entry cost c_i is $\bar{c}$ with probability p and $\underline{c}$ with probability $1 - p$. Each c_i is private information to Firm i. Firm i has payoff $\Pi^m - c_i$ if i is the only firm to enter and $\Pi^d - c_i$ if both firms enter. Not entering yields a payoff 0. Assume that $\Pi^m > \bar{c} > \Pi^d > \underline{c} > 0$.

   (a) Formulate the game as a Bayesian game.

   (b) For what values of p there is a symmetric Bayes Nash equilibrium where both players enter if and only if their cost is $\underline{c}$?

   (c) Suppose $p\Pi^m + (1 - p)\Pi^d > \bar{c}$. Find all Bayes Nash equilibria of the game.

   (d) Now keeping the assumption that $p\Pi^m + (1 - p)\Pi^d > \bar{c}$, suppose that Firm 1 enters first and Firm 2 decides about entry after observing Firm 1's decision. Find all weak Perfect Bayesian equilibria.

3. Consider the following stage game G (P1 is the row player and P2 is the column player):

	L	R
T	$1, -2$	$3, -2$
M	$6, 0$	$4, 1$
D	$5, 2$	$0, 3$

(a) Find all Nash equilibria of the stage game.

(b) According to the minmax folk theorem, what payoffs are possible to attain in a subgame perfect equilibrium when G is infinitely repeated and the common discount factor δ goes to 1? (You are allowed to answer with a picture.)

(c) Draw an automaton that describes the following strategy profile for the infinitely many times repeated game: P1 plays M in the first period and if either (M, L) or (T, R) was played in the previous period, otherwise P1 plays T; P2 plays L in the first period and if either (M, L) or (T, R) was played in the previous period, otherwise P2 plays R.

(d) Is the strategy profile in part (c) a subgame perfect equilibrium for some $\delta \in (0, 1)$?

4. Consider the following (fictional) situation. Denmark has one vacant slot for the Olympic games in cycling. There are two candidates who may get chosen and the final decision is based on a qualification race that takes place before the Olympics in the same season. Should the athletes time their best performance for the qualification race or for the Olympics?

You are allowed to combine sub questions but then you need to state clearly which sub questions you are answering together.

(a) Define a game that describes the situation.

(b) Point out what assumptions you have made in part (a).

(c) What would be a suitable solution concept to solve the game in part (a)? Argue why.

(d) Write down the equations that characterize a solution (this means that a strategy profile that satisfies all of them is a solution).

(e) Either solve the game OR discuss what you would expect to happen in the game (the latter means writing a few sentences where you describe the main tradeoff).

(f) Interpret your results (write a few sentences).